

## UG Semester V

### Paper 9: Numerical Analysis

Credit: 4

T:04

#### Course Outcomes:

1. Some numerical methods to find the zeroes of nonlinear functions of a single variable and solution of a system of linear equations, up to a certain given level of precision.
2. Interpolation techniques to compute the values for a tabulated function at points not in the table.
3. Applications of numerical differentiation and integration to convert differential equations into difference equations for numerical solutions.

#### Unit I

Solution of equations: bisection, Secant, Regular Falsi, Newton Raphson's method, Newton's method for multiple roots, Interpolation, Lagrange and Hermite interpolation, Difference schemes, Divided differences, Interpolation formula using differences.

#### Unit II

Numerical differentiation, Numerical Quadrature: Newton Cotes Formulas, Gaussian Quadrature Formulas, System of Linear equations: Direct method for solving systems of linear equations (Gauss elimination, LU Decomposition, Cholesky Decomposition), Iterative methods (Jacobi, Gauss Seidel, Relaxation methods). The algebraic Eigen value problem: Jacobi's method, Givens method, Power method.

#### Unit III

Numerical solution of Ordinary differential equations: Euler method, single step methods, Runge-Kutta method, Multi-step methods: Milne-Simpson method, Types of approximation: Last Square polynomial approximation, Uniform approximation, Chebyshev polynomial approximation.

#### Unit IV

Difference Equations and their solutions, Shooting method and Difference equation method for solving Linear second order differential equation with boundary conditions of first, second and third type.

#### References

#### Text Books:

1. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI
2. Numerical Methods for Engineering and scientific computation by M. K. Jain, S.R.K. Iyengar & R.K. Jain.

Suggested Readings:

3. Kandasamy P. & et Al., Numerical Methods, S. Chand & Co.

Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

<https://openlearninglibrary.mit/edu/courses>

<http://heecontent.upsdc.gov.in/SearchContent.aspx>

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### **Paper 10: Analysis**

Credit: 4

T:04

Course Outcomes:

1. Understand the basic concepts of metric spaces.
2. Know the concepts such as open balls, closed balls, compactness, connectedness etc.
3. Understand the significance of differentiability of complex valued functions leading to the understanding of Cauchy-Riemann equations.
4. Evaluate the contour integrals and understand the role of Cauchy-Goursat theorem and the Cauchy integral formula.
5. Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues and apply Cauchy Residue theorem to evaluate integrals.

Unit I

Definition and examples of metric spaces, Neighbourhoods, Interior points, Limit Points, Open and closed sets, Convergent and Cauchy sequences, Completeness, Cantor's intersection theorem.

Uniform convergence of sequences and series of functions, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Power series.

Unit II

Stereographic projection, Continuity and Differentiability of complex functions, Analytic functions, Cauchy Riemann equations, Harmonic functions.

### Unit III

Complex integration, Cauchy-Goursat theorem, Cauchy's Integral formula, Formulae for first, second and nth derivatives, Cauchy's Inequality, Liouville's Theorem, Elementary functions, Mapping by elementary functions, conformal mapping.

### Unit IV

Taylor and Laurent Series, Absolute and uniform convergence of Power series, Residues and Poles, Residue theorem, Zeros and poles of order m, Evaluation of improper real integrals, Definite integrals involving sines and cosines.

#### References:

#### Text books:

1. Mathematical Analysis by Shanti Narain.
2. Complex variable and applications by Brown & Churchill.

#### Suggested Readings:

3. Magnus Robert, Fundamental Mathematical Analysis, Springer Undergraduate Mathematics Series

#### Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

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### **Paper 11 A: Integral & Partial Differential Equations**

Credit: 4

T:04

#### Course Outcomes:

1. Describe different types of Linear integral equations and partial differential equations for the impart knowledge of formulation of practical problems of applied mathematics.
2. Understand the theoretical basic behavior of different types of arising problems such as Fredholm, Volterra, Singular, Hilbert and Cauchy integral equations.
3. Explain the foundations of various problems related to Wave, Laplace and Diffusion equations by the method of separation of variables.
4. Deal with problems in applied mathematics, theoretical mechanics and mathematical physics and engineering.

## Unit I

Origin of first order partial differential equations. Partial differential equations of the first order and degree one, Lagrange's solution, Partial differential equation of first order and degree greater than one. Cauchy's method of characteristic, Charpit's method of solution, Surfaces orthogonal to the given system of surfaces.

## Unit II

Origin of second order PDE, Solution of partial differential equations of the second and higher order with constant coefficients, Classification of linear partial differential equations of second order, Solution of second order partial differential equations with variable coefficients, Monge's method of solution, Cauchy's problem for Homogenous wave equation, Properties of Harmonic function, Methods of separation of variable for solving Laplace, wave and diffusion equations.

## UNIT III

Linear Integral Equations-Definition and Classification of conditions, Special kinds of Kernels, Eigen values and Eigen functions, Convolution integral, Inner product, Integral equations with separable Kernels. Reduction to a system of algebraic equations.

## UNIT IV

Fredholm alternative, Fredholm Theorem, Fredholm alternative theorem, Approximate method, Method of successive approximations, Iterative scheme. Solution of Fredholm and Volterra integral equation, Results about resolvent Kernel.

References:

Text Books:

1. I.N. Sneddon: Elements of Partial Differential Equations, Mc -Graw Hill, 1988.
2. Ram P. Kanwal, Linear Integral Equations (2nd ed.), Birkhäuser, Boston.

Suggested Readings:

3. T. Amarnath: An Elementary Course in Partial Differential Equations, Narosa Publishing House, New Delhi, 2005.
4. Tyn Myint U: Partial Differential Equations of Mathematical Physics, Elsevier Publications.

## Web References:

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## **Paper 11B: Discrete Mathematics**

Credit: 4

T: 04

### Course Outcomes:

1. Lattices and their types.
2. Boolean algebra, switching circuits and their applications.
3. Graphs, their types and its applications in study of shortest path algorithms.
4. Display familiarity with the mathematical models which are the integral part of the hardware and software of computer science.
5. Elaborate and expand their understanding of the tools helpful in the implementation of circuit design, AI algorithms and compiler construction.

### Unit I

Propositional Logic- Proposition logic, basic logic, logical connectives, truth tables, tautologies, contradiction, normal forms (conjunctive and disjunctive), modus ponens and modus tollens, validity, predicate logic, universal and existential quantification, proof by implication, converse, inverse contrapositive, contradiction, direct proof by using truth table.

### Unit II

Boolean Algebra- Basic definitions, Sum of products and products of sums, duality principle, Boolean functions, Logic gates and Karnaugh maps. Lattice, Duality, types of lattices, sublattices, bounded lattices, distributive lattices, complemented lattices, modular lattices, join irreducible elements.

### Unit III

Combinatorics- Inclusion- exclusion, recurrence relations (nth order recurrence relation with constant coefficients, Homogeneous recurrence relations, Inhomogeneous recurrence relations), generating function (closed form expression, properties of G.F., solution of recurrence relations using G.F. solution of combinatorial problem using G.F.)

## Unit IV

Finite Automata- Basic concepts of automation theory, Deterministic Finite Automation (DFA), transition function, transition table, Non Deterministic Finite Automata (NFA), Mealy and Moore machine, Minimization of finite automation.

### References:

#### Text books:

1. Discrete Mathematics by C. L.Liu.
2. Discrete Mathematics with computer application by Trembley and Manohar.
3. Mendelson, Elliott: Introduction to Mathematical Logic, Chapman & Hall, 1997
4. John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman: Introduction to Automata Theory, Languages and Computation, Pearson Education, 2000

#### Suggested Readings:

5. Arnold B. H.: Logic and Boolean Algebra, Prentice Hall, 1962
6. K. H. Rosen: Discrete Mathematics and its applications, MGH 1999

#### Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

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## **Paper 11C: Number Theory**

Credit: 4

T:04

### Course Outcomes:

1. To have knowledge of primes, congruences, quadratic residues and primitive roots.
2. Solving Diophantine equations.
3. Derive generating functions and recurrence relations.

## UNIT I

Divisibility; Euclidean algorithm; primes; congruences; Fermat's theorem, Euler's theorem and Wilson's theorem; Fermat's quotients and their

elementary consequences; solutions of congruences; Chinese remainder theorem; Euler's phi-function. Congruences

## UNIT II

Congruence modulo powers of prime; primitive roots and their existence; quadratic residues; Legendre symbol, Gauss' lemma about Legendre symbol; quadratic reciprocity law; proofs of various formulations; Jacobi symbol.

## UNIT III

Diophantine Equations, Solutions of  $ax + by = c$ ,  $x^n + y^n = z^n$ ; properties of Pythagorean triples; sums of two, four and five squares; assorted examples of diophantine equations.

## UNIT IV

Generating Functions and Recurrence Relations, Generating Function Models, calculating coefficient of generating functions, Partitions, Exponential Generating Functions, A Summation Method. Recurrence Relations: Recurrence Relation Models, Divide and conquer Relations, Solution of Linear, Recurrence Relations, Solution of Inhomogeneous Recurrence Relations, Solutions with Generating Functions.

References:

Text Books:

1. Niven, I., Zuckerman, H. S. and Montgomery, H. L. (2003) An Int. to the Theory of Numbers (6th edition) John Wiley and sons, Inc., New York.
2. Burton, D. M. (2002) Elementary Number Theory (4th edition) Universal Book Stall, New Delhi.
3. Balakrishnan, V. K. (1996) Introductory Discrete Mathematics, Dover Publications.

Suggested Readings

4. Balakrishnan, V. K. (1994) Schaum's Outline of Theory and Problems of Combinatorics Including Concepts of Graph Theory, Schaum's Outline.

Web References:

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## **Internship / Term Assignment**

Credit :04

## UG Semester VI

### Paper 12: Advanced Algebra

Credit: 4

T:04

Course Outcomes:

1. Give the structure of an abelian group of a given order.
2. Construct the splitting field extension of a given polynomial.
3. Understand the interplay of group theory and field theory.
4. Determine the minimal polynomial of an algebraic element.

Unit I

Series of groups, Schreier theorem, Jordan Holder theorem, solvable groups, Nilpotent groups, Insolvability of  $S_n$  for  $n > 5$ ,

Unit II

Finite Abelian groups, primary decomposition theorem, basis theorem, fundamental theorem of finite Abelian group, elementary divisors and invariant factors,

Unit III

Field extensions: finite extension, finitely generated extension, algebraic extension, simple extension, transcendental extension, finite field.

Unit IV

Splitting field, algebraically closed field, normal extension, separable extension, primitive element theorem. Galois theory- Galois group, Galois extension, Fundamental theorem of Galois theory, Artin's theorem, Fundamental theorem of algebra (Algebraic Proof)

References:

Text Books:

1. V. Sahai & V. Bist: Algebra, Fourth Edition, Narosa.
2. J. A. Gallian, Contemporary Abstract Algebra, 4th edition, Narosa
3. DJS Robinson, An Introduction to Abstract Algebra, Hindustan Book Agency.

Suggested Readings:

4. J. B. Fraleigh: A first course in Abstract algebra, Narosa
5. S. Lang: Algebra, Addison Wesley.

Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

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**Paper 13: Differential Geometry & Tensor Analysis**

Credit: 4

T:04

Course Outcomes:

1. Explain the concept of differentiable geometry.
2. Understand the concepts of tensors in differentiable geometry.
3. Apply various concept of differential calculus in tensors.

Unit I

Local theory of curves-Space curves, Examples, Plane Curves, tangent and normal and binormal, Osculating Plane, normal plane and rectifying plane, Helices, Serret-Frenet apparatus, contact between curve and surfaces, tangent surfaces, involutes and evolutes of curves, Bertrand curves, Intrinsic equations, fundamental existence theorem for space curves.

Unit II

Metric-first fundamental form and arc length, Direction coefficients, families of curves, intrinsic properties, geodesics, canonical geodesic equations, normal properties of geodesics, geodesics curvature, Gauss-Bonnet theorem, Gaussian curvature, normal curvature, Meusnier's theorem, mean curvature, Gaussian curvature, umbilic points, lines of curvature, Rodrigue's formula, Euler's theorem.

Unit III

Tensor algebra: Vector spaces, the dual spaces, tensor product of vector spaces, transformation formulae, contraction, special tensor, inner product, associated tensor.

Tensor Analysis: Contravariant and covariant vectors and tensors, Mixed tensors, Symmetric and skew-symmetric tensors, Algebra of tensors, Contraction and inner product, Quotient theorem, Reciprocal tensors, Christoffel's symbols, Covariant differentiation.

## Unit IV

Gradient of scalars, Divergence of a contra-variant vector, covariant vector and conservative vectors, Laplacian of an invariant, curl of a covariant vector, irrotational vector, Riemannian space, Riemannian curvatures and their properties, Ricci tensor, and scalar curvature, Einstein space and Einstein tensor, Geodesics.

### References:

#### Text books:

1. T.J. Willmore, An introduction to Differential Geometry, Dover Publication 2012.
2. S.Lang., Fundamentals of Differential Geometry; Springer, 1999.
3. B. O'Neil, Elementary Differential Geometry, 2nd Edition, Academic press, 2006.
4. R.S. Mishra, A Course in Tensors with Application to Riemannian Geometry, Pothishala 1988.

### Suggested Readings:

5. David C. Kay, Tensor Analysis, Schaum's Outline series McGraw Hill 1988.

### Web References:

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## **Paper 14 A: Advanced Differential Equations**

Credit: 4

Course Outcomes:

T:04

1. Solve the system of 1st order differential equations, 2nd order differential equations, nth order differential equations, oscillatory equation, stability and unstability of linear and non-linear system of equations.
2. Conceptualize Green's functions and nature of critical points.
3. Prove advanced understanding of topics in applied mathematics, computational physics etc.

## Unit I

Linear System- Introduction, properties of linear homogeneous systems, Abel-Liouville formula, Periodic linear System, Floquet's theorem, Solution of nth order linear homogeneous equation with variable coefficients.

## Unit II

Inhomogeneous linear system, nth order linear non-homogeneous equation with variable coefficients, Hurwitz's theorem, Non-linear system, Volterra's prey & predator equation, Non linear equations: Autonomous system.

## Unit III

The phase plane & its phenomena, types of critical points & Stability, Critical points & stability for linear system, stability by Liapunov's direct method. Green function, Construction of Green functions, Green function of homogeneous and non-homogeneous end conditions, Strum Liouville systems.

## Unit IV

Second order differential equation: Introduction, Preliminary results, Boundedness of solutions, Oscillatory equation, number of zeroes, Pruffer's transformation, Strum theorem, Strum's comparison theorem.

References:

Text Books:

1. G. F. Simmons: Differential Equation, Tata McGraw-Hill
2. B. Rai, D. P. Chaudhary, H.I. Freedman: A course in Ordinary Differential Equations, Narosa Publishing House.
3. S. L. Ross: Differential Equations, Wiley Indian, 2004

Suggested Readings:

4. E. A. Coddington: An Introduction to Ordinary Differential Equations

Web References:

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## Paper 14 B: Operations Research

Credit: 4

T:04

### Course Outcomes:

1. Be able to understand the application of OR and frame a LP Problem with solution
2. Be able to build and solve Transportation and Assignment problems using appropriate method.
3. Be able to design and solve simple models of CPM and queuing to improve decision making and develop critical thinking and objective analysis of decision problems.
4. to take best course of action out of several alternative courses for the purpose of achieving objectives by applying game theory and sequencing models.

### Unit I

Linear programming problems, Slack and surplus variables, Statement of general Linear programming Problems, Standard and matrix forms of linear programming problem, Basic feasible solution.

### Unit II

Convex sets, Fundamental theorem of linear programming, Simplex method. Artificial variables, Big-M method, Two- phase method, Revised simplex method.

### Unit III

Resolution of degeneracy, Duality in linear programming problems, Dual simplex method, Primal-dual relation analysis, integer programming.

### UNIT IV

Transportation problems, assignment problems, Queuing Theory, Markov Chains, PERT and CPM, Optimization and constrained Optimization using Langrange's Multiplier.

### References:

#### Text books:

1. Hamdy A. Taha, Operations Research: An Introduction, 10th Edition, Pearson
2. Kanti Swaroop, P. K. Gupta, Manmohan, Operations Research, Sultan Chand

Suggested Readings:

3. G. Hadley, Linear Programming

Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

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**Minor Project**

Credit :04

## UG SEMESTER VII

### Paper 15: Topology

Credit: 4

T:04

Course Outcomes:

1. Define and illustrate the concept of topological spaces and continuous functions,
2. Illustrate the concept of limit point, dense sets, interior, exterior, boundary points.
3. Identify and understand bases, sub-bases and different type of spaces like Lindelof, Separable, and their properties.

Unit I

Countable and uncountable sets, Schroeder-Bernstein theorem, Cantor's Theorem, Cantor's Sets, Cantor's continuum hypothesis, Zorn Lemma, Well ordering principle.

Topological spaces: Definitions and Examples, open base and open subbase for a topology, Lindelof theorem, limit points, closure, interior; Continuous functions, Homeomorphisms; relative topology, Metric Topology, Product Topology, Weak topology, The function algebras  $C(X,R)$  and  $C(X,C)$ .

Unit II

Compact spaces, Heine Borel theorem, product of spaces, Tychonoff theorem, generalized Heine Borel theorem, locally compact spaces, compactness for metric spaces, Ascoli's theorem.

Unit III

Separation Axioms:  $T_1$  and Hausdorff spaces, completely regular and normal spaces, Urysohn's lemma; Tietze extension theorem. Urysohn's imbedding theorem; Stone Cech compactification.

UNIT IV

Connected spaces, the components of a space, totally disconnected space, locally connected space,

References

Text Books:

1. G.F. Simmons: Introduction to Topology and Modern Analysis, Mc-Graw Hill Int. Book Company
2. J. R. Munkres: Topology - A first course, Prentice hall India Pvt. Ltd.

Suggested Readings:

3. J. L. Kelley: General Topology. Van Nostrand. Reinhold Co., New York 1995

Web References:

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### **Paper 16: Fluid Mechanics**

Credit: 4

T:04

Course Outcomes:

1. understand the concept of fluid and their classification, models and approaches to study the fluid flow.
2. formulate mass and momentum conservation principle and obtain solution for non viscous flow.
3. know potential theorems, minimum energy theorem and circulation theorem.
4. understand two dimensional motion, circle theorem and Blasius theorem.

Unit I

Types of fluids, Continuum hypothesis, Lagrangian and Eulerian method of describing fluid motion, Motion of Fluid element: Translation, Rotation and Deformation. Stream lines, Path lines and streak lines. Material derivative. Acceleration of a fluid particle in Cartesian, Cylindrical Polar and Spherical Polar Coordinates. Vorticity Vector, Vortex Lines, Rotational and Irrotational motion of fluid, Rotational velocity, Velocity Potential, Boundary surface, Boundary condition.

Unit II

Reynold transport theorem. Principle of conservation of Mass-Equation of continuity (By Lagrangian and Eulerian method). Equation of Continuity in different coordinate systems. Body force and Surface force. Euler's equation of motion-conservation of momentum, Bernoulli's Equation, Energy Equation, Impulsive effects.

Unit III

Irrotational motion in two dimensions: Stream function, Physical significance of stream function, Sinks, Doublets and their images in two dimension, Complex Velocity Potential. Sources, Milne-Thompson circle theorem, Vortex,

Vortex motion, Image of Vortex, Kelvin Circulation Theorem, Complex potential due to Vortex, Kirchhoff vortex Theorem, Blasius Theorem and Kutta-Joukowski Theorem.

#### Unit IV

Irrotational motion produced by motion of circular cylinders in an infinite mass of liquid, Liquid Streaming past circular cylinder, Kinetic energy of liquid, Motion of sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere, Axis-Symmetric flow, Stoke's function.

#### References:

#### Text Books:

1. Frank Chorlton: Text Book of Fluid Dynamics, C.B.S. Publishers, Delhi.
2. Z.U.A. Warsi: Fluid Dynamics, Theoretical and Computational Approaches, C.R.C. Press
3. S.W. Yuan: Foundation of Fluid Mechanics, Prentice Hall of India Pvt. Ltd. New Delhi
4. N. Curle and H J Davies: Modern fluid dynamics

#### Suggested Readings:

5. G. K. Batchelor: An Introduction to Fluid Dynamics. Cambridge University Press. London.
6. R.W. Fox, P.J. Pritchard and A.T. McDonald: Introduction to Fluid Mechanics, Seventh Edition, John Wiley & Sons, 2009.

#### Web References:

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### **Paper 17: Differential Geometry of Manifolds**

Credit: 4

T:04

#### Course Outcomes:

1. Elaborate the concept of differentiable manifolds and their examples.
2. Clarify the concepts of vector fields, tangent vectors & tangent spaces in a manifold.
3. Apply various concepts of differential calculus to the settings of abstract set called manifold.

4. Use Riemannian metric on a given manifold to find the various types of curvatures with emphasis on the surface/ types of manifold.
5. Bring out different connections on Riemannian manifold and its properties.
6. Calculate curvature tensor & tensors of respective connections.

#### Unit I

Definition and examples of differentiable manifolds, Tangent vectors, Tangent Spaces, Vector fields and their examples, Jacobian map. Immersions and submersions, Diffeomorphism and their examples, Curve in a manifold, Integral curves and their examples, Distributions, Hypersurface of  $\mathbb{R}^n$ , Sub-manifolds.

#### Unit II

Standard connection on  $\mathbb{R}^n$ , Covariant derivative, Sphere map, Weierstrass map, Gauss equation, the Gauss curvature equation and Codazzi-Mainardi equations.

#### Unit III

Invariant viewpoint, Cartan view point, coordinate view point, Difference Tensor of two connections, Torsion and curvature tensors.

#### Unit IV

Riemannian Manifolds, Length and distance in Riemannian manifolds, Riemannian connection and curvature, Curves in Riemannian manifolds, Submanifolds of Riemannian manifolds.

#### References:

##### Text Books:

1. N.J. Hicks: Notes on Differential Geometry, D. Van Nostrand, 1965.
2. U. C. De., A. A. Shaikh: Differential Geometry of Manifolds, Narosa Publishing House.

##### Suggested Readings:

3. Y. Matsushima: Differentiable Manifolds, Marcel Dekker, INC. New York, 1972.

##### Web References:

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## Paper 18: Complex Analysis

Credit: 4

T:04

### Course Outcomes:

1. Understand the topics of Complex Analysis needed to pursue research in pure mathematics.
2. Understand the properties of maximum modulus of a Complex valued function and the results based on that property.
3. Develop manipulation skills in the use of Rouché's theorem and Argument Principle.
4. Show knowledge of Gamma and Zeta functions with their properties and relationships.
5. Understand the Harmonic functions defined on a disc and concerned results.
6. Make factorization of entire functions having infinite number of zeros.

### Unit I

Maximum Modulus Theorem, Schwarz's Lemma, Minimum Modulus Theorem, Hadamard's three circle theorem, automorphism of the unit disk. Convergence of sequences and series of complex numbers, absolute convergence. Uniform convergence of sequence and series of functions, Cauchy's criterion, Weierstrass's M-test, analytic convergence theorem. Absolute and uniform convergence of power series, integration and differentiation of power series, radius of convergence.

### Unit II

Zeros of holomorphic functions, Open Mapping Theorem, Inverse Function Theorem. Index of a closed path, meromorphic functions, argument principle, Rouché's theorem, residue at the point at infinity, indentation around a branch point and the branch cut, summation of series.

### Unit III

Function spaces: Hurwitz theorem, Infinite products, Weierstrass factorization theorem, Mittag-Leffler's theorem, Gamma functions and its properties, Riemann's Zeta function.

### UNIT IV

Uniqueness of direct analytic continuation, Power series method of analytic continuation, Natural boundary, Schwarz's reflection principle, Harmonic Functions, Mean value property for harmonic functions, Harnack's inequality, Poisson formula, Jensen's formula, Poisson-Jensen's formula, Convex functions, Hadamard's three circle theorem as a convexity theorem, Canonical products, Hadamard's factorization theorem, order of entire functions.

References:

Text Books:

1. J. V. Deshpande: Complex Analysis, Tata McGraw-Hill Publishing Company Limited, New Delhi
2. E. C. Titchmarsh: Theory of functions, Oxford University Press
3. John B. Conway: Functions of one complex variables, Springer International

Suggested Readings:

4. R.V. Churchill, J.W. Brown, Complex Variables and Applications, McGraw Hill.

Web References:

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### **Paper 19 A: Module Theory**

Credit: 4

T:04

Course Outcomes:

1. Identify cyclic modules, simple modules, finitely generated modules etc.
2. Find a basis of a free module.
3. Use the basis to describe module homomorphisms.
4. Describe the structure of a finitely generated module over a PID.

Unit I

Modules-Definition and examples, simple modules, submodules, Module Homomorphisms, Quotient modules, torsion free and torsion modules, direct sum of modules.

Unit II

Exact sequences, Short exact sequence, split exact sequences, Five-lemma, free modules, modules over division rings are free modules, invariant rank property.

### Unit III

Free modules over PID's, Invariant factor theorem for submodules, Finitely generated modules over PID, Chain of invariant ideals, Fundamental structure theorem for finitely generated module over a PID,

### Unit IV

Projective and injective modules, Divisible group.

#### References:

#### Text Books:

1. V.Sahai and V. Bist: Algebra, Fourth Edition, Narosa.
2. I.B.S. Passi and I.S. Luther: Algebra, Volume 3 Modules, Narosa

#### Suggested Readings:

3. S. Lang: Algebra, Addison Wesley.

#### Web References:

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## **Paper 19 B: Measure Theory & Integration**

Credit: 4

T:04

#### Course Outcomes:

1. Display understanding of the essential foundations of important aspect of mathematical analysis.
2. Explain the measurability of a set of real numbers and measurable functions.
3. Differentiate between the Riemann integral and the Lebesgue integral.
4. Apply the Measure theory and theory of the integral in other branches of pure and applied mathematics.

### Unit I

Algebra of sets, countable sets, Cantor set, Borel sets, outer measure of a set and its properties. Measurable sets. Lebesgue measure, a non-measurable set.

Measurable functions and their properties. Concept of almost everywhere. Littlewood's three principles.

#### Unit II

The Lebesgue integration of bounded function over a set of finite measure, the Lebesgue, Bounded convergence theorem, the integral of a non-negative function, Fatou's Lemma, Monotone convergence theorem, the general Lebesgue integral, Lebesgue convergence theorem.

#### Unit III

Differentiation of monotone functions, Vitali's Lemma, the four derivatives, the differentiation theorem. Functions of bounded variation, Differentiation of an integral. Absolute continuity.

#### Unit IV

Inequalities and the  $L_p$  Spaces: The  $L_p$  Spaces, convex functions, Jensen's inequality, the inequalities of Holder and Minkowski, completeness of  $L_p(\mu)$ . Convergence in measure, almost uniform convergence.

#### References:

##### Text Books:

1. H.L. Royden: Real Analysis, Pearson Prentice Hall
2. G.de Barra: Measure Theory and Integration, Wiley Eastern Ltd.

##### Suggested Readings:

3. Taylor, Measure Theory and Integration, American Mathematical Soc., 2006

##### Web References:

Digital platforms web links: NPTEL/SWAYAM/ MOOCS/Openstax.org

<https://openlearninglibrary.mit.edu/courses>

<http://heecontent.upsdc.gov.in/SearchContent.aspx>

<https://www.lkouniv.ac.in/en/article/e-content-faculty-of-science>

## **Research Methodology**

Credit: 4

T:04

Course outcomes:

This course is designed to enable students to:

1. Identify and discuss the role and importance of research.
2. Identify and discuss the issues and concepts salient to the research process.
3. Identify and discuss the complex issues inherent in selecting a research problem, selecting an appropriate research design, and implementing a research project.
4. Identify and discuss the concepts and procedures of sampling, data collection, analysis and reporting
5. Read, comprehend and explain research article and writing a research article.

### **UNIT I**

#### **Research Formulation and Design**

Motivation and objectives – Research methods vs. Methodology. Steps of research, Types of research – Descriptive vs. Analytical, Applied vs. Fundamental, Quantitative vs. Qualitative, Conceptual vs. Empirical, concept of applied and basic research process, criteria of good research. Defining and formulating the research problem, selecting the problem, necessity of defining the problem, importance of literature review in defining a problem, literature review-primary and secondary sources, reviews, monograph, patents, research databases, web as a source, searching the web.

### **UNIT II**

#### **Data measurement and Data Analysis**

Measurement: Concept of measurement, Problems in measurement in research – Validity and Reliability. Levels of measurement – Nominal, Ordinal, Interval, Ratio. Observation and collection of data, methods of data collection, sampling methods, data processing and analysis strategies and tools, data analysis, hypothesis testing.

### **UNIT III**

#### **Soft Computing**

Computer and its role in research, some mathematical software like MATLAB, R etc. and their application in research. Software for paper formatting like LaTeX/MS Office.

## **UNIT IV**

### **Research Ethics and Report Writing**

Significance of Report Writing, Different Steps in Writing Report, Layout of the Research Report, Ethical issues related to publishing, Plagiarism, Software for detection of Plagiarism, publishing a research article.

#### REFERENCES:

1. Kothari, C.R., 1990. Research Methodology: Methods and Techniques. New Age International.
2. Garg, B.L., Karadia, R., Agarwal, F. and Agarwal, U.K., 2002. An introduction to Research Methodology, RBSA Publishers

#### Suggested Readings:

3. A Manual for Writers of Research Papers, Theses, by Kate L. Turabian, Wayne C Booth, Gregory G. Colomb.

#### Web references:

<https://www.classcentral.com/course/swayam-research-methodology-17760>  
<http://users.cla.umn.edu/~nwaller/math.htm>

## **UG SEMESTER VIII**

### **Major Project**

Credit :24